5 Dynamic Phase Diagram for a Periodically Driven Kinetic Square-lattice Ising Ferromagnet: Finite-size Scaling Evidence for the Absence of a Tri-critical Point

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Abstract. We discuss the subtle finite-size effects of the dynamic phase transition (DPT) in a two-dimensional kinetic Ising ferromagnet driven by an oscillating external field. We present computational and analytical evidence that there is *no* finite-temperature tri-critical point in the dynamic phase diagram of this model. This contrasts with earlier claims [1–3] for the existence of a tri-critical point in this model. Careful finite-size scaling analysis of Monte Carlo simulations reveals that the negative dip of the Binder cumulant and the corresponding multi-peak order-parameter distribution (often characteristic of a first-order transition) are merely finite-size effects in this case. When the DPT prevails in the infinite-system limit, it is always continuous. The misleading finite-size effects are related to the stochastic nature of the underlying metastable decay for "small" systems, which exhibit stochastic resonance.

5.1 Introduction

Hysteresis occurs in bistable systems coupled to a periodically changing external field. The intrinsic time scale of these systems is given by the metastable lifetime $\langle\tau\rangle$, the mean time spent in the metastable well. In this paper we use magnetic language, in which the order parameter is the magnetization m, and its conjugate field is the external magnetic field H. The metastable lifetime is defined as the average first-passage time to zero magnetization after a sudden field reversal.

An important aspect of hysteresis in bistable systems is the dynamic competition between two time scales: the half-period of the external field $t_{1/2}$ (proportional to the inverse of the driving frequency) and the average metastable lifetime $\langle\tau\rangle$ of the system. For low frequencies, the time-dependent magnetization oscillates about zero in synchrony with the external field (symmetric dynamic phase). For high frequencies, however, the system does not have time to switch during one half-period, and the magnetization oscillates about one or the other of its degenerate zero-field values (asymmetric dynamic phase). This symmetry breaking and the corresponding dynamic phase transition (DPT) has attracted considerable attention in the last decade [1–7].

The intimate connection between the underlying decay modes [8] and the response of *spatially extended* kinetic Ising ferromagnets in periodic applied fields has been stated and demonstrated several times [5,6,9]. Nevertheless, there still appears to be some confusion [3] about the correct interpretation of simulation results, especially when it comes to the careful analysis of the somewhat subtle finite-size effects. Therefore, we briefly summarize the theoretical framework needed to understand the underlying metastable decay mechanisms and their consequences for the periodic response, and we present new Monte Carlo (MC) results that support our theoretical picture.

5.2 Modes of Metastable Decay and Finite-size Effects of the Periodic Response

The simulations presented here were performed on a kinetic Ising ferromagnet on a two-dimensional square lattice of linear size L, described by the Hamiltonian $\mathcal{H} = -J\sum_{\langle ij\rangle} s_i s_j - H(t)\sum_{i=1}^{L^2} s_i$, where $s_i = \pm 1$ is the state of the ith spin, $J > 0$ is the ferromagnetic coupling constant, $\sum_{\langle ij\rangle}$ runs over all nearest-neighbor pairs, and $H(t)$ is an oscillating, spatially uniform applied field. We use a square-wave field, which has computational advantages over a sinusoidal field, while it does not change the main characteristics of the system response. The dynamic used is the single-spin-flip Glauber algorithm with updates at randomly chosen sites [9]. Time is reported in units of MC steps per spin (MCSS). The density conjugate to $H(t)$ is the magnetization per site, $m(t) = (1/L^2)\sum_{i=1}^{L^2} s_i(t)$. The temperature T is kept below the equilibrium zero-field critical temperature T_c. In each half-period (for not too large field amplitudes) the system then either attempts to escape the metastable phase through the nucleation of droplets, or it simply stays in the stable phase if escape was unsuccessful during the preceding half-period. The dynamic order parameter is the period-averaged magnetization $Q = 1/(2t_{1/2})\oint m(t)\mathrm{d}t$, where the beginning of the period is chosen at a time when $H(t)$ changes sign.

For sufficiently large systems (see quantitative statement below) the system escapes from the metastable phase through the nucleation of *many* droplets [multi-droplet (MD) regime]. Consequently, the time-dependent system magnetization is self-averaging. If $\langle\tau(T,H)\rangle \ll t_{1/2}$, the magnetization follows the external field in each half-period, the system relaxes to a symmetric limit cycle, and the probability density for Q, $P(Q)$, is sharply peaked at zero. On the other hand, for $\langle\tau(T,H)\rangle \gg t_{1/2}$ the probability of leaving a free-energy well is very low, and the system reaches an asymmetric limit cycle (with occasional switches between the two equivalent symmetry-broken dynamic phases). In this case $P(Q)$ becomes bimodal with sharp peaks near ± 1. This breaking of the limit-cycle symmetry and the associated DPT have been carefully analyzed [5,6] using finite-size scaling techniques borrowed from equilibrium theory [9,11]. In terms of the dimensionless half-period, $\Theta = t_{1/2}/\langle\tau(T,H)\rangle$, the DPT should occur at a critical value $\Theta_c \sim \mathcal{O}(1)$.

For any finite system, however, the metastable decay mode changes to the nucleation and growth of a *single* droplet [single-droplet (SD) regime] at sufficiently low temperatures. Due to the *stochastic* nature of the nucleation of a single droplet, the corresponding response in the presence of a periodic field is drastically different: the system exhibits stochastic resonance (SR) [9].

The crossover from MD to SD decay can be understood using the standard theory of homogeneous nucleation [8]. The average time between nucleation events (the nucleation time) is obtained as $t_n(L,T,H) = [L^d I(T,H)]^{-1}$, where $I(T,H)$ is the nucleation rate per unit volume. It can be expressed in terms of the free energy of the critical droplet, $F(T,H)$, as $I(T,H) = C(T,H)^{-1}\exp(-F(T,H)/T)$. Here $F(T,H)$ and the prefactor $C(H,T)$ can be obtained from nucleation theory [8]. The growth time t_g is defined as the time it takes for a supercritical droplet to grow to fill half the system volume. Assuming constant radial velocity it can be written as $t_g(L,T,H) = L/\{[2\Omega_d(T)]^{1/d}v(T,H)\}$, where $\Omega_d(T)$ is a dimension- and temperature-dependent shape factor [8] and $v(T,H)$ is the propagation velocity of the field-driven droplet surface [12]. For $t_n \ll t_g$ many droplets nucleate while those nucleated shortly after the field reversal are still growing. For $t_n \gg t_g$ the first droplet to nucleate eventually will fill the system on its own. The crossover from the MD to the SD regime is formed by the dynamic spinodal (DSP) [8] and can be estimated by equating t_n and t_g. This yields an implicit equation for the temperature corresponding to the DSP as a function of L and H,

$$T_{\rm DSP} = \frac{F(T_{\rm DSP},H)}{(d+1)\ln L - \ln\{C(T_{\rm DSP},H)[2\Omega_d(T_{\rm DSP})]^{1/d}v(T_{\rm DSP},H)\}}\ . \quad (5.1)$$

An estimate for $T_{\rm DSP}(L)$ at $H = 2.0J$, obtained by numerically solving (5.1), is shown as a dashed curve in Fig. 5.1a. In the large-system limit $T_{\rm DSP}$ logarithmically approaches zero. Reducing T at fixed $t_{1/2}$ and H results in fundamentally different behavior for small and large systems.

In a small system, the underlying decay mode changes from MD to SD as T is reduced below$T_{\rm DSP}$, and as a result SR is exhibited. The SR behavior is characterized by a multi-peak $P(Q)$, including two sharp peaks at ± 1 and one centered about zero. Further, the Binder cumulant, $U_L = 1-\langle Q^4\rangle_L/(3\langle Q^2\rangle_L^2)$ [9] attains a negative minimum before changing sign again to approach its ordered-phase value of 2/3 (Fig. 5.1b). Reducing T further, magnetization switches become increasingly rare, and the system essentially becomes frozen

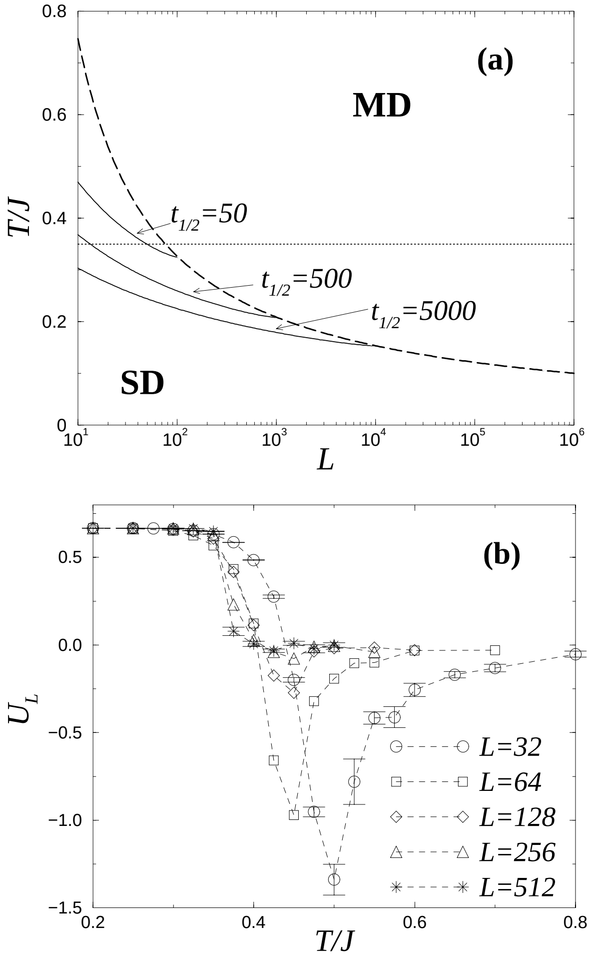

Fig. 5.1. (**a**) Metastable decay modes and crossovers in the dynamic phase diagram at $H = 2.0J$. The dashed curve is the dynamic spinodal, $T_{\mathrm{DSP}}(L)$, separating the multi-droplet (MD) from the single-droplet (SD) regime. The solid curves correspond to $T_\times(L)$ for various half-periods $t_{1/2}$, indicated by the arrows (in MCSS). The horizontal dotted line indicates the temperature $T/J = 0.35$, where the DPT occurs for $t_{1/2} = 50\,\mathrm{MCSS}$ in the large-L limit. Here the lifetime for large systems is $\langle\tau\rangle \approx 50\,\mathrm{MCSS}$. (**b**) The finite-size effects in the Binder cumulant at $H = 2.0J$ for $t_{1/2} = 50\,\mathrm{MCSS}$

in one or the other free-energy well ($U_L \approx 2/3$), giving rise to yet another crossover line. This line is estimated by assuming that SD switching is a Poisson process and requiring that the probability that the magnetization does not change sign during a half-period in which it is opposite to the field should equal 1/2. This again leads to an implicit equation for the corresponding crossover temperature,

$$T_\times = \frac{F(T_\times, H)}{d \ln L + \ln \left[\frac{1}{C(T_\times, H) \ln 2} \left(t_{1/2} - \frac{L}{[2\Omega_d(T_\times)]^{1/d} v(T_\times, H)} \right) \right]} . \quad (5.2)$$

Estimates of $T_\times(t_{1/2}, L)$ at $H = 2.0J$ for different $t_{1/2}$, obtained by numerically solving (5.2), are shown as solid curves in Fig. 5.1a. For each value of $t_{1/2}$, the curves representing $T_{\rm DSP}(L)$ and $T_\times(t_{1/2}, L)$ form the border of a wedge-shaped region in which SR is observed. The multi-peaked $P(Q)$ and the negative minimum of the Binder cumulant predicted above for the SR regime between $T_{\rm DSP}(L)$ and $T_\times(t_{1/2}, L)$ have previously been observed in MC [3]. However, these effects were misinterpreted as signs of a first-order phase transition, separated from a line of continuous DPT by a tri-critical point [1–3]. Negative cumulants and a corresponding minimum were observed for small systems ($L \leq 128$) in our simulations with $t_{1/2} = 50$ MCSS (Fig. 5.1b).

The behavior as T is lowered at constant H and $t_{1/2}$ is qualitatively different for larger systems, beyond the value of L where the curve representing $T_\times$ for that particular value of $t_{1/2}$ meets the curve representing $T_{\rm DSP}$. For these large systems, the temperature that corresponds to the DPT is significantly larger than $T_{\rm DSP}$, and the system undergoes a continuous DPT at this higher temperature. Below the DPT temperature the symmetry of the order-parameter oscillations is broken, and the Binder cumulant attains its ordered-phase value of 2/3. The corresponding behavior of the cumulant, characteristic of a second-order phase transition, is seen for $L = 256$ and 512 in Fig. 5.1b. We note that it is our choice of $t_{1/2} = 50$ MCSS that enables us to probe both the SR and DPT regimes in simulations with reasonably-sized systems – larger values of $t_{1/2}$ (as used in [3]) would require very large systems to avoid the SR regime.

5.3 Summary

While stochastic resonance is an important feature of our system with possible scientific applications, it does not survive in the large-system limit. Our results provide estimates for the corresponding crossovers and clarify the dynamic phase diagram of the periodically driven kinetic Ising ferromagnet and other spatially extended bistable systems which belong to the same dynamic universality class [7,13]. In particular, our results indicate that effects that were previously thought to indicate the existence of a tri-critical point separating lines of second-order and first-order dynamic phase transitions, are merely finite-size effects associated with stochastic resonance in relatively small systems.

Acknowledgments

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